

Students Learning to See Ten as a Unit to Solve Multidigit Addition Word Problems

This story is a part of the series:

What's Next? Stories of Teachers Engaging in Collaborative Inquiry Focused on Using Student Thinking to Inform Instructional Decisions

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What's Next?

Stories of teachers engaging in collaborative inquiry focused on using student thinking to inform instructional decisions

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Introduction

This lesson details how a teacher used first graders' solutions to a word problem involving multidigit addition to advance student understanding of how the base-ten structure of numbers can be used to add multidigit numbers. This lesson was developed collaboratively by a group of teachers participating in a professional development experience, after the group used individual interviews to learn about the strategies the first-grade students were already using to solve addition and subtraction word problems involving two-digit numbers.

Relevant Florida Mathematics Standards

MAFS.1.NBT.3.4 Add within 100, including adding a two-digit number and a one-digit number and adding a two-digit number and a multiple of 10, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used. Understand that in adding two-digit numbers, one adds tens and tens, ones and ones, and sometimes it is necessary to compose a ten.

MAFS.1.NB.2.2 Understand that the two digits of a two-digit number represent amounts of tens and ones.

- a. 10 can be thought of as a bundle of ten ones—called a "ten."
- b. The numbers from 11 to 19 are composed of a ten and one, two, three, four, five, six, seven, eight, or nine ones.
- c. The numbers 10, 20, 30, 40, 50, 60, 70, 80, or 90 refer to one, two, three, four, five, six, seven, eight, or nine tens (and 0 ones).
- d. Decompose two-digit numbers in multiple ways (e.g., 64 can be decomposed into 6 tens and 4 ones or into 5 tens and 14 ones).

Background Information

Chapter seven of Children's Mathematics: Cognitively Guided Instruction (Carpenter et al., 2015) offers useful background information on how children use base-ten thinking to varying degrees when encouraged to generate their own ways of solving problems involving multidigit numbers. A trajectory of student thinking is presented, illustrating how students develop and use increasingly sophisticated understanding of base-ten concepts in their strategies.

Carpenter, T. P, Fennema, E., Franke, M. L., Levi, L., & Empson, S. B. (2015) *Children's Mathematics: Cognitively Guided Instruction*. Portsmouth, NH. Heinemann.

Analyzing Student Thinking

In the context of a professional development experience, a group of teachers conducted brief individual interviews with first-grade students in a single class. The purpose of the interviews was to gain insight into the degree to which the first grade students were using base-ten concepts in multidigit addition and subtraction word problems and use that insight to design a lesson for those students on that day that may help advance their understanding.

In interviews lasting about 15 minutes each, students were asked to solve two to four of the following word problems using their own strategies:

Problem A. Pete has 20 rocks. Juan gives him 24 more rocks. How many rocks does Pete have now?

Problem B. Tylesha has 32 books. Her grandma gives her 25 more books. How many books does Tylesha have now?

Problem C. Mr. Jones had 40 cupcakes. He gave 20 cupcakes to the students in his class. How many cupcakes does Mr. Jones have now?

Problem D. Maria had 35 jellybeans. Her dad gave her 27 more jellybeans. How many jellybeans does Maria have now?

All interviews began with Problem A, but after

observing how a student responded to Problem A, the interviewer had the flexibility to pose items out of order, to skip items, and to adjust the numbers if they were perceived to be too difficult for the student. The teachers agreed that, if possible, they would try to pose Problem D to all students so they could observe how students added five and seven.

Interviewers attempted to support students' understanding of the problem, as needed, and asked probing questions to get a better understanding of students' mathematical thinking. As each problem was posed, students were encouraged to pay close attention to the problem situation and to generate their own way of solving the problem using tools (e.g., manipulatives, fingers, writing materials) or mental strategies. Because they were interested in what students understood and could do on their own on this day, interviewers took care to avoid teaching or showing students how to solve the problem.

After the interviews, teachers agreed that most students in the class were making sense of the problem situations and using viable strategies. The teachers focused on discussing students' specific strategies for solving Problem D. In particular, the teachers discussed the degree to which various students' strategies used knowledge of base-ten concepts, and they classified students' strategies into categories along a continuum of <u>abstraction. The</u> four major strategies categories are described below. Figure 1 details how the teachers classified the strategies observed among students in the class.

Multidigit Computation Strategies¹

A student who uses a *direct modeling with ones* strategy represents each multidigit number in the problem as a set of ones using manipulatives or pictures to model the story in the problem and then counts the objects or pictures to determine the answer. For example, for the Problem D, a student might draw a set of 35 circles and a set of 27 circles and then count all the circles, either starting at one or counting on from 35 or 27.

A student who uses a *counting by ones* strategy does not physically represent all of the numbers in the problem. Fingers, objects, or tally marks are often used to keep track of the number of counts. For example, in solving Problem D, a student might start at 35 and count forward by ones for 27 counts, keeping track of the counts on fingers or with cubes or tally marks.

A student who uses a *counting by tens* strategy represents each multidigit number in the problem using manipulatives or pictures that reflect the base-ten structure of our number system (e.g., with base-ten blocks or base-ten pictures). Then the student counts the objects or pictures to determine the answer. For example, in solving Problem D, a student might draw base-ten pictures

1 The descriptions of strategies presented here are the current descriptions used by our team, and we consider them to be fluid, as our understanding of these ideas continues to evolve. For a more detailed discussion of these terms, consider reading Carpenter et al. (2015).

			Invented	
Direct modeling	Counting by ones	Counting by tens	algorithm	Other
lsaac	Xiamar	Kayla*	Jonathon**	Mia
Aryanna	Kamaya	Paige		Adrian
		Steven		
		Maria		
		Jason*		
		Christian		
		Nicco*		

*These students were placed in the *counting by tens* category because they exhibited *counting by tens* in some aspect of their solution, but their strategies were noted to use ten as a unit only inconsistently, because tens were sometimes directly modeled or counted by ones.

**Jonathon's invented algorithm strategy involved both combining like units and incrementing.

Figure 1. Strategies used by students in a first-grade class to solve Problem D.

representing 35 as three line segments and five dots and 27 as two line segments and seven dots. The student might then count toward the total by first counting the line segments by tens, "10, 20, 30, 40, 50" and then count the dots by ones, "51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62." Another student might build the 35 and 27 in a similar manner with base-ten blocks but then count some or all of the individual units on the base-ten blocks to determine the answer.

A student who uses an *incrementing* strategy determines the answer by increasing partial sums. For example, in solving Problem D, a student might decompose the 27 into 10 + 10 + 5 + 2, so it can be added to the 35 in manageable parts: $35 + 10 = 45 \rightarrow 45 + 10 = 55 \rightarrow 55 + 5 = 60 \rightarrow 60 + 2 = 62$.

A student who uses a *combining like units* strategy operates on the tens and ones separately, and the partial sums are then combined to yield a final result. For example, in solving Problem D, a student might decompose 35 into 30 + 5 and 27 into 20 + 7, add the tens (i.e., 30 + 20 = 50) and then add the ones separately (i.e., 5 + 7 = 12). Last, the student would operate on these partial sums to determine the answer (i.e., 50 + 12 = 62).

A student who uses a *compensation* strategy adds or subtracts the numbers by adjusting one number to compensate for changes made in another number. For example, in solving Problem D, a student might initially substitute 30 for the 27 to ease the difficulty of the computation. After mentally adding 35 + 30 = 65, the student would subtract 3 from the sum (i.e., 65 - 3 = 62) to compensate for the difference between 27 and 30.

Overall, the teachers noticed that students' approaches to Problem D offered evidence that some students were thinking in terms of grouping by tens in the base-ten system, whereas others appeared to be thinking about the quantities in the problem purely as sets of ones without making use of ten as a unit. Among those student strategies classified as using *counting by tens*, several modeled the quantities with tens and

then counted sets of ten by ones. On the basis of these observations, the teachers conjectured that the class would benefit from a lesson designed to highlight the opportunities to improve efficiency by using ten as a unit within their strategies. They established the following learning goal:

Students will come to understand how thinking of ten as a unit can be used to solve two-digit addition problems.

Planning for the Lesson

In efforts to work toward the established goal, the teachers decided to develop a lesson in which the students would examine and articulate how different student-generated strategies for two-digit addition problems used ten as a unit. They decided to focus the lesson on class discussion of the Problem D, which the students had already solved in the interview. (Maria had 35 jellybeans. Her dad gave her 27 more jellybeans. How many jellybeans does Maria have now?) Then they would have the teacher pose a new problem with a similar context and numbers to offer students the opportunity to experiment with ideas brought forward in the class discussion.

Given their desire to make explicit the opportunities to use base-ten thinking for solving two-digit addition problems, the teachers thought carefully about which particular student strategies observed for Problem D would be most useful for the class to discuss. They decided to focus on having students make sense of Steven's counting by tens strategy with base-ten blocks and then Jonathon's invented algorithm strategy (see both students' strategies in Figure 2).

Steven's counting by tens strategy was selected, because all of the students in this first-grade class were familiar with base-ten blocks and would probably be able to make sense of how Steven initially represented 35 and 27. Comparing baseten blocks to base-ten pictures, the teachers noted that each unit cube is visible on the base-ten blocks in a way that is not present in typical baseten pictures (in which tens are depicted with line

Steven's counting by tens strategy	Jonathon's invented algorithm strategy
Steven built 35 and 27 using base-ten blocks. He counted the blocks by tens and ones as he built	Jonathon solved the problem mentally and described his strategy as follows:
each number (e.g., for 35, he said 10, 20, 30, 31,	
32, 33, 34, 35).	"First I did 30 and 20 to get 50. Then 5
	[more] is 55, and 7 [more] is (tracking on
To determine the total, Steven counted all of ten rods by tens and then all of the unit cubes by	fingers) 56, 57, 58, 59, 60, 61, 62. It is 62."
ones, "10, 20, 30, 40, 50, 51, 52, 53, 54, 55, 56,	The teachers generated this symbolic
57, 58, 59, 60, 61, 62."	representation of Jonathon's strategy:
	30 + 20 = 50
	50 + 5 = 55
	55 + 7 = 62

Figure 2. Details	of two students	' strategies for	Problem D.
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segments). The teachers determined that the students who would benefit most from discussion of Steven's strategy were those who had used a *direct modeling* or *counting by ones* strategy and also the students who had used *counting by tens* but then counted the tens by ones to determine the answer. They therefore decided that questions about Steven's strategy would be directed to these students as much as possible in hopes that they might later experiment to a greater extent with a strategy involving units of ten.

Jonathon's invented algorithm strategy was selected for focus in discussion, because it offers an abstraction of Steven's counting by tens strategy. The teachers conjectured that making sense of this strategy would be a logical next step for students who were already counting by tens. They decided first to work with the class to figure out how Jonathon's strategy could be notated and then to work collaboratively to compare and contrast the solution to Steven's strategy.

Because the teachers intended to design the lesson such that it would advance the understanding of students who demonstrated different levels of base-ten thinking in the interview, they planned to direct questions about a given strategy to the students they perceived to be ready to use that strategy. Steven's strategy was selected for focus to help move students using *direct modeling* or *counting by ones* strategies to making sense of the counting by tens strategy. Jonathon's strategy was selected to help move students already using a counting by tens strategy to consider a more abstract approach. In addition, the teachers planned to differentiate instruction by directing students to solve the new word problem (at the end of the lesson) using a strategy of their own choice. By allowing students to solve problems in their own ways while also encouraging experimentation with increasingly abstract strategies, the teachers intended to create an environment in which all students would progress along the strategy continuum.

Lesson Plan

The lesson plan that follows was developed to work toward the following goal:

Students will come to understand how thinking of ten as a unit can be used to solve two-digit addition problems.

In service of this goal, this lesson plan details a plan to orchestrate a class discussion lasting approximately 20 minutes focused on examining student work for a problem that students have already solved. To support teacher efforts to implement this lesson, the lesson plan includes activities to complete before the lesson and during the lesson. All of these activities can be completed in a single lesson, but doing so requires the teacher

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to analyze student responses and formulate a plan for class discussion during student work time.

Before the lesson

- Have students solve Problem D (i.e., Maria had 35 jellybeans. Her dad gave her 27 more jellybeans. How many jellybeans does Maria have now?) in accordance with the following expectations:
 - a. Every student should work independently to solve the problem in a way that makes sense to him or her, using mental strategies, manipulatives (e.g., base-ten blocks, snap cubes) pictures, or written numerals.
 - b. Students should record their thinking with pictures, numbers, words, or some combination or show how they used manipulatives to solve the problem.
 - c. If pictures or manipulatives were used, students should record the answer to the problem in numerals.
- 2. During or after students solve the problem, examine students' solution strategies in relation to the continuum of strategies introduced in Phase 2 (i.e., *direct modeling, counting by ones, counting by tens, invented algorithms*).
- 3. Identify and sequence two to four student solutions to use in class discussion that will help to advance student understanding of how thinking of ten as a unit can be used to solve two-digit addition problems. Possibly useful is to identify students who have used the following strategies, depending on the overall profile of your class:
 - a. A student who has used pictures of individual jellybeans organized in clear sets of ten or perhaps a student who has represented the quantities in ten frames (a *counting by tens* strategy). These strategies clearly depict the individual jellybeans, which is helpful to students who are *counting by ones*, while also making visible how the ones can

This observation suggests that students will benefit from continued opportunities to gain greater comfort with modeling and counting by ten as a unit. be organized and counted as sets of ten.

- b. A student who has represented the quantities with base-ten blocks. The physical base-ten blocks also make visible the individual units within each ten but in a slightly more abstract way than pictures of sets of ten or ten frames.
- c. A student who has represented the quantities with base-ten quick pictures (use of line segments to represent tens and dots to represent ones). This strategy can be useful for introducing the idea that one need not necessarily see all of the individual units within the tens.
- d. A student who has used an *invented algorithm* that is closely related to the ways students have used *counting by tens* strategies. Comparing this strategy to an already examined *counting by tens* strategy helps to build a bridge between *direct modeling* and more abstract strategies that do not involve physical or pictorial modeling.
- e. A student who has used an *invented algorithm* that does not align as closely with the *counting by tens* strategy. These strategies offer useful extension for students who have moved beyond *direct modeling*.

During the lesson

- 1. Commend the students for their work on Problem D and explain that you have selected a few students' solutions for the class to examine together. State explicitly that, as we look at each solution, everyone must try to understand what the student was thinking and to ask questions when unsure.
- 2. Post a written version of the problem and have the class read it aloud. Then engage students in recalling the important details of the problem, either as a whole class or with shoulder partners.
- 3. Guide students to examine, one at a time, the

...many students in the class were much more comfortable thinking of the quantities as sets of ones and counting up by ones (rather than taking advantage of opportunities to count by tens). two to four solutions chosen for focus in class discussion. Sequence the sharing of strategies from less to more sophisticated, with the goal of providing opportunities for students to make sense of increasingly abstract strategies. Through the discussion, attend to the following when appropriate and applicable:

- a. Provide opportunities for the students to explain their understanding of the solution strategies of others in relation to the problem context.
 - i. Have a student who has used manipulatives share the steps followed, and ask the class to explain why each step was taken, including how it is related to the word problem.
 - ii. Present student work for all to see (on document camera, board, or on carpet) and have the class make conjectures about what the student did. Then the student whose work is being shared can verify or refute the conjectures.
 - iii. Ask questions to help students "notice" important aspects of each solution. For example: How does this solution show the 35 jellybeans that Maria already had? the 27 jellybeans her dad gave her? How do we know that 35 jellybeans are represented here?
- b. Take advantage of opportunities to highlight how base-ten relationships were and can be used in the focal student solutions:
 - i. When a strategy involves *direct modeling* of the quantities with tens or *counting by tens*, prompt the class to consider why the student did that. Why did [student] use three base-ten rods and five unit cubes to represent 35? How did she know it was 35 without counting by ones?
 - ii. When a students' strategy could have used ten as a unit but did not, encourage the class to consider how they might

use their knowledge of ten. For example, commonly, some students will represent 35 and 27 with base-ten blocks (using knowledge of base-ten) but then determine the answer by using counting on by ones, starting at 35 and pointing at each unit of the representation of 27 saying, "36, 37, 38 . . . 61, 62." After this strategy is shared, prompt the class to consider whether some other way to count might be more efficient.

- iii. Prompt students to compare and contrast *direct modeling* strategies that ten as a unit with strategies that do not. Challenge students to locate the opportunities to notice tens within the *counting by ones* strategies.
- iv. Prompt students to compare and contrast *invented algorithm* strategies with similar *counting by tens* strategies. When *invented algorithms* are the focus of class discussion, be sure to make visible mathematical notation that details the steps of the strategy.
- c. Choose students to provide explanations or answer questions with the goal of supporting individual students with advancing their personal understanding of base-ten thinking and how it might be used to solve this particular problem.
 - i. Invite students who have struggled to interpret the problem to answer questions that probe the relationship between the problem context and students' solutions.
 - ii. Invite students who use *counting by ones* to explain the most concrete *counting by tens* strategy.
 - iii. Invite students who have used *counting* by tens strategies that involved representing with tens but *counting* by ones to explain how *counting* by tens can be used in the solutions of others.

- iv. Invite students who have used *counting* by tens to explain their classmate's *invented algorithm* strategies.
- 4. Provide an opportunity for students to experiment independently with ideas brought forward in discussion by solving a similar follow-up problem (as an exit activity). Explicitly encourage students to consider using the strategies and ideas of focus in discussion, and if time allows, have students solve the problem in two ways. A possible follow-up problem: Maria has 24 jellybeans. Dad gave her 17 more. How many does she have now?

Reflection

What happened in the classroom

After prompting students to recall how they solved the problem, the teacher noticed that many of the students had used base-ten blocks to solve it. She then invited Steven to share with the class his strategy with base-ten blocks. Steven built 35 and 27 with the base-ten blocks. Then he counted the ten rods by tens, "10, 20, 30, 40, 50," and continued counting the unit cubes by ones, "51, 52, 53...60, 61, 62." Next the teacher prompted the class to explain what Steven had done. After multiple students contributed to explaining parts of Steven's strategy, the teacher had the whole class work together to count by tens and ones to verify that the two sets were composed of 35 and 27. She prompted students to consider why they could count the ten rods in the models by skip-counting by tens, drawing out of the class the justification that each ten rod is the same as ten unit cubes or ten ones.

At this point, Kayla suggested to the teacher and to the class a different way to use the base-ten blocks. With teacher permission, Kayla moved to the central position on the carpet, facing her classmates, and built 35 with base-ten block rods and units and 27 with base-ten block unit cubes. She then said, "You can use 35 with tens and ones and then add 27 ones." The teacher asked Kayla why she thought Steven did it his way (representing both numbers with tens and ones), and

Kayla asserted that she thought he just got mixed up and that it would have been easier if he had used all ones for the 27. Steven responded that he thought it was easier to use tens and ones for both numbers because, ". . . counting by tens is easier than counting so many ones." Around the carpet, opinion was mixed about which strategy made the problem easier. Several students agreed with Kayla's assertion that it is easier to "see" the 27 as 27 ones, whereas others focused on the ease of counting by tens in Steven's strategy. The teacher acknowledged Steven and Kayla with a thinking expression and a nodding head and said to the class, "We are all going to have to keep thinking about this . . . when counting by tens makes solving a problem easier."

The teacher then directed Steven and Kayla to go back to their sitting spaces on the carpet perimeter, and she shifted the focus to having students draw connection between Kayla's model and the context of the word problem. Pointing at the set of 27 unit cubes in Kayla's model, the teacher prompted students to discuss, "What are these ones in our story?" and "Where in Kayla's model are the jellybeans that Maria had at the beginning of the story?" The class was slow to respond to these questions, so the teacher allowed several students to offer ideas until the class seemed to reach consensus on how the parts of Kayla's strategy corresponded to the word problem. The teacher then refocused the class on the difference between Steven's and Kayla's way of counting their models. She noted that Kayla started at 35 and counted the unit cubes by ones, and she elicited help from the class to recall how Steven had counted the base-ten blocks in his model by tens and ones.

Next the teacher invited Jonathon to share his (*invented algorithm*) mental strategy. Jonathon quietly told the class, "I counted up from 50. I counted five and then seven." The teacher wrote on the board 50 + 5 + 7 = 62. After a few moments of letting the class attempt to process what Jonathon had shared, the teacher led students to think about how they could use their fingers to start at 50 and count up five and then seven. Then she asked, "Why did Jonathon start with 50? No

50 appears in our story. Where did the 50 come from?" Many students in the class appeared unsure, but Kamaya tentatively suggested that the 50 came from the numbers 35 and 27. A few other students started to conjecture that the three and two from 35 and 27 together made 50. The teacher directed students to look at Steven's base-ten blocks solution. More confidently, Kamaya picked up the three ten rods and two ten rods in Stephen's solution and exclaimed that they were the 50. The teacher revoiced the idea raised by Kamaya and the others saying, "Three tens and two tens are five tens or 50." She underlined the digit three and two in the equation 35 + 27 = 62, and recorded underneath the equation 30 + 20 =50. The teacher then directed students to spend a few moments explaining to their shoulder partners where the 50 in Jonathon's solution came from. Finally, the teacher led the class to connect the five and seven in Jonathon's solution to the sets of unit cubes in Stephen's counting by tens solution with base-ten blocks.

At this point, the time left in the lesson was short; so the teacher directed students to return quickly to their desks and solve a problem with the same context and different numbers: Maria had 24 jellybeans. Her dad gave her 17 more. How many does she have now? Around the room many students employed the same strategy they had used on the original problem, but a few attempted to use a new strategy that mirrored one of the strategies shared in discussion.

Planning for Future Lessons

The teacher had planned to focus discussion in this lesson on two strategies: Stephen's counting by tens strategy and Jonathon's invented algorithm strategy. During the lesson, Kayla raised the idea that her approach involving direct modeling the 35 with tens and ones and the 27 with all ones was easier. In order for the class to be able to engage with the idea that Kayla was raising, significant time, not previously planned, was allocated to discussing Kayla's strategy and then evaluating her assertion that it was easier. Although the inclusion of Kayla's strategy made the discussion longer than the teacher had intended, the exchange that accompanied discussion of it offered significant insight into how the students were thinking. It revealed that many students in the class were much more comfortable thinking of the quantities as sets of ones and counting up by ones (rather than taking advantage of opportunities to count by tens). Contrasting Steven's and Kayla's approaches to direct modeling and counting seemed helpful to the students.

Student strategies for the follow-up problem revealed that many students preferred Kayla's way of thinking about the 27 as 27 ones. This observation suggests that students will benefit from continued opportunities to gain greater comfort with modeling and counting by ten as a unit. One idea raised later by the teacher was that having all students solve a similar problem, with the expectation that they solve it in at least two different ways, might be beneficial. Using such an approach, could lead students both to work with a comfortable strategy and then to experiment with a less comfortable one.

What's Next?

Stories of teachers engaging in collaborative inquiry focused on using student thinking to inform instructional decisions

What's Next? is a collection of stories documenting professional development experiences shared by elementary teachers working collaboratively to study the complex process of teaching and learning mathematics. Each story in the collection describes practicing teachers studying the thinking processes of real students and using what they learn about those students to make decisions and try to help advance those students' understanding on that day.

The teachers in each story start by learning about how individual students are solving a set of mathematics problems. They use this freshly gathered knowledge of student thinking to develop nearterm learning goals for students and a lesson plan tailored to specific students on that specific day. One of the teachers implements the planned lesson while the other teachers observe in real time. The teachers then gather to discuss and reflect on their observations and insights. In these lessons, the practice of teaching is slowed way down. The stories tell of teachers who are studying student thinking and using that information to plan and implement instructional decisions at a pace that is much slower than it occurs in daily practice. The stories in this collection also depict many aspects in common with formative assessment and lesson study, both of which are a process and not an outcome.

The stories depict real situations that occurred in real time and include both successes and shortcomings. We hope that the stories may be studied and discussed by interested educators so that the lessons and ideas experiences of these teachers and instructional coaches may contribute to additional learning and sharing among other interested teachers.

Learn more about these and other stories at http://www.teachingisproblemsolving.org/

